Assessing the effects of streamline curvature on the aerodynamics of circulation control airfoil

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RANS simulation of a circulation control airfoil was performed at a chord based Reynolds number of $0.49 \times 10^6$ using an incompressible solver in OpenFOAM. In this flow configuration, a tangential jet is blown over a thick, rounded trailing edge to delay separation using the Coanda effect. Highly curved, recirculation regions are seen to form near the trailing edge. To investigate the streamline curvature effects, curvature corrections from both bifurcation approach and modified coefficients approach were applied to the SST $k$-$\omega$ turbulence model. The predicted radial velocity profiles along the Coanda surface, the pressure coefficient and the lift coefficient are compared with the LES data. Some improvements observed in using the curvature corrections are discussed.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$c$</td>
<td>Airfoil chord length</td>
</tr>
<tr>
<td>$A$</td>
<td>Planform area</td>
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<tr>
<td>$U_j$</td>
<td>Jet velocity</td>
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<tr>
<td>$U_\infty$</td>
<td>Freestream velocity</td>
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<tr>
<td>$q$</td>
<td>Dynamic pressure, $(1/2\rho U^2)$</td>
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<tr>
<td>$L$</td>
<td>Lift force</td>
</tr>
<tr>
<td>$C_j$</td>
<td>Jet momentum coefficient</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Pressure coefficient, $(p - p_\infty)/(1/2\rho U_\infty^2)$</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Lift coefficient, $L/(1/2\rho A U_\infty^2)$</td>
</tr>
<tr>
<td>$\theta_{sep}$</td>
<td>Jet separation angle</td>
</tr>
<tr>
<td>TKE</td>
<td>Turbulent Kinetic Energy</td>
</tr>
<tr>
<td>SST</td>
<td>Shear Stress Transport</td>
</tr>
<tr>
<td>$P$</td>
<td>Production term in the TKE transport equation</td>
</tr>
<tr>
<td>$D_k$</td>
<td>Dissipation term in the TKE transport equation</td>
</tr>
<tr>
<td>$CD_\omega$</td>
<td>Cross-diffusion term in the $\omega$ transport equation</td>
</tr>
<tr>
<td>$D_\omega$</td>
<td>Dissipation term in the $\omega$ transport equation</td>
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I. Introduction

Circulation control can enhance the lift of the airfoils by making use of the so-called “Coanda effect” — the tendency of a pressurized gas to adhere to the curved surface. A tangential jet is blown over a thick, curved trailing edge to delay separation and thus generating higher lift. This configuration has been under extensive study for more than two decades. Circulation control technology has been leveraged across several other

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Novak et al.\textsuperscript{14} conducted experiments on circulation control airfoil at a Reynolds number of $0.98 \times 10^6$ and at two jet momentum coefficients of $C_j = 0.03$ and $C_j = 0.1$. Several numerical investigations to understand the detailed flow physics using LES and DNS have been reported in the literature (Slomski et al.,\textsuperscript{17} Hahn and Shariff,\textsuperscript{8} Madhavan and Rogers\textsuperscript{10}). Nishino and Shariff\textsuperscript{12} studied the effect of windtunnel side walls on the numerical simulations. Swanson and Rumsey\textsuperscript{18} have conducted RANS simulations on this configuration. They found that streamline curvature effects can play a significant role especially at higher jet momentum coefficients. Without curvature corrections, they showed that both Spalart-Allmaras and SST $k - \omega$ models predict a jet wrap around the airfoil which is not physical. When a curvature correction is used, physically reasonable results were reported. Pfingsten et al.,\textsuperscript{15} Arolla and Durbin\textsuperscript{1} have also performed RANS simulations on this configuration with similar conclusions.

Recently, Englar et al.\textsuperscript{6} designed an airfoil and conducted experiments on this new configuration. But, no measurements have been reported so far. Nishino et al.\textsuperscript{13} performed LES on Englar airfoil to understand the detailed flow physics. They obtained good agreement with the experiments at a jet momentum coefficient of $C_j = 0.044$, but the agreement was poor at $C_j = 0.12$. Rumsey and Nishino\textsuperscript{16} have studied the effect of streamline curvature through RANS simulations with different turbulence models. They showed improvements in predicting the location of the jet separation over the Coanda surface when the streamline curvature effects are accounted. However, the lift coefficient predictions were found to be approximately $10 - 15\%$ higher than LES at a higher jet momentum coefficient. The physical reason for this discrepancy was not clear.

In this work, we investigate the effect of streamline curvature with two approaches for modeling curvature effects: bifurcation approach and modified coefficients approach (see Durbin,\textsuperscript{1} Arolla and Durbin\textsuperscript{2,3} for more details). The objective is to compare the aerodynamic characteristics predicted by these models and to assess the effects of streamline curvature.

I.A. Concept of circulation control

From a fundamental aerodynamics point of view, circulation control can be understood using Kutta-Joukowski’s theorem: “Lift is directly proportional to circulation”. Lift per unit span, denoted $L'$ on the airfoil is given by

$$L' = \rho_{\infty} U_{\infty} \Gamma$$

where $\Gamma$ is the circulation which is given by

$$\Gamma \equiv \oint_A \mathbf{V} \cdot d\mathbf{s}$$

where $A$ is any closed curve around the airfoil (shown in Figure 1). Hence, any increase in circulation results in increased lift. The name “Circulation control” is, perhaps, derived from this viewpoint. In the high lift airfoil configuration, the farther the flow remains attached over the trailing edge of the airfoil, the higher is the circulation and hence the lift. The circulation is “controlled” by adjusting the blowing coefficient.

To understand the lift generation in the configuration considered, two computations were performed: with zero blowing and with a finite amount of blowing. The resulting streamtraces around the airfoil are plotted in Figure 1. When a finite amount of blowing was used, the forward stagnation point moved down and, the flow separation over the Coanda surface was delayed. Lift generated by finite blowing is higher because the separation got delayed. Hence, one can think of it as “separation control”. So, accurate prediction of the separation characteristics and the pressure distribution is critical in understanding the aerodynamic characteristics.

I.B. Role of turbulence

Due to sufficiently high Reynolds numbers, the external flow around the airfoil is fully turbulent. When the suction surface turbulent boundary layer interacts with the wall jet from the plenum chamber, the jet
transitions to turbulence. Therefore, the transition characteristics of the jet over the Coanda surface are assumed to be less important.\textsuperscript{13}

To understand the aerodynamic characteristics of the circulation control airfoil, it is important to understand: the interaction of the turbulent wall jet with the suction surface boundary layer, the dynamics of flow separation and reattachment over the curved surface, and the spreading of the jet sheet downstream of the airfoil. A turbulent wall jet can be thought of as a two-layer shear flow: in an inner layer, the flow exhibits similarities in structure with the conventional turbulent boundary layer; and in an outer layer, the shear-layer character is more like free-shear flow. For the turbulent wall jet over a convex surface, the turbulent transport is enhanced in the outer region and diminished in the inner region (Lauder and Rodi\textsuperscript{9}). The Coanda surface is convexly curved and hence predicting both stabilizing and destabilizing effects on turbulence determines the separation location and the jet spreading rate. But, the most widely used two-equation models cannot adequately predict the curvature effects. In this work, we use two approaches of modeling curvature effects proposed in our earlier work\textsuperscript{3} discussed in the following section.

II. Turbulence modeling

We used the SST variant of $k-\omega$ (Menter\textsuperscript{11}) as the base turbulence model. It has the following form:

$$\frac{\partial k}{\partial t} + u_j \frac{\partial k}{\partial x_j} = P - D_k + \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_T \frac{\sigma_k}{\sigma} \right) \frac{\partial k}{\partial x_j} \right]$$ (3)

where $D_k = \beta^* k\omega$.

$$\frac{\partial \omega}{\partial t} + u_j \frac{\partial \omega}{\partial x_j} = \frac{\gamma}{\nu_T} P - D_\omega + \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_T \frac{\sigma_\omega}{\sigma} \right) \frac{\partial \omega}{\partial x_j} \right] + CD_\omega$$ (4)

where $D_\omega = \beta^2 \omega^2$. The eddy viscosity, with no accounting for curvature effect is $\nu_T = C_\mu k/\omega$ with $C_\mu=1.0$. The two approaches of modeling streamline curvature effects are briefly outlined below.

II.A. Bifurcation approach

Arolla and Durbin\textsuperscript{3} proposed a correction to the eddy viscosity coefficient based on the bifurcation analysis of the second moment closures (SMCs) in rotating homogeneous shear flow. It has the following functional form:

$$C^*_\mu = C_\mu \left( \alpha_1 (|\eta_3| - \eta_3) + \sqrt{1 - \min(\alpha_2 \eta_3, 0.99)} \right)^{-1}$$
where the model constants obtained are: $\alpha_1 = 0.04645$, $\alpha_2 = 0.25$. The correction factor is clipped from above at 2.5.

II.B. Modified coefficients approach

A modified coefficients model for the production term of the $\omega$-equation (see Arolla and Durbin), $P_\omega = \gamma F_{rc} S^2$ where

$$F_{rc} = (1.0 + \alpha_1 |\eta_3| + 3\alpha_1 \eta_3)$$

(5)

The correction is clipped above at 2.5 and below at 0. The model coefficient, $\alpha_1 = -0.2$.

The invariants used in the models are then computed as:

$$\eta_1 = S_{ij}S_{ij}T^2; \quad \eta_2 = \Omega^{mod}_{ij}\Omega^{mod}_{ij}T^2; \quad \eta_3 = \eta_1 - \eta_2$$

The following time scale is used:

$$T_1 = \frac{1}{\beta^*\omega}; \quad T_2 = 6\sqrt{\frac{\nu}{\beta^*k\omega}}; \quad T_3 = (T_1^nT_2)^{\frac{n+1}{n+2}}; \quad T = \max(T_1, T_3) \quad \text{with} \quad n = 1.625$$

The strain rate tensor and rotation rate tensor are given by:

$$S_{ij} = \frac{1}{2}(\partial_j U_i + \partial_i U_j); \quad \Omega^{mod}_{ij} = \Omega^A_{ij} + (C_r - 1)W^A_{ij}$$

(6)

where $\Omega^A_{ij} = \Omega^{rel}_{ij} + \Omega^F_{ij}$ with $\Omega^{rel}_{ij} = \frac{1}{2}(\partial_j U_i + \partial_i U_j)$ and $\Omega^F_{ij} = -\epsilon_{ijk}\omega_k$. The Spalart-Shur tensor is defined as:

$$\Omega^{SS} \equiv \Omega^F - \frac{S.D.S - D.S.S}{2|S|^2}$$

(7)

In 2D, $W^A_{ij} = \Omega^{SS}_{ij}$. In 3D, this is changed to $W^A_{jk} = \Omega^F_{jk} - \epsilon_{ijk}w_i$ in which

$$w_i = II_S X_{ij}(\Omega^F_{pq}\epsilon_{pq} - \Omega^{SS}_{rs} \epsilon_{rs})$$

$$X_{ij} = \frac{II^2_S \delta_{ij} + 12III_S S_{ij} + 6II_S S_{ik} S_{kj}}{2II^2_S - 12III^2_S}$$

where $II_S = S_{ij}S_{ji}$ and $III_S = S_{ij}S_{jk}S_{ki}$. The coefficient $C_r = 2$ is used for both the models.

III. RANS simulations: Results and Discussion

The above curvature models were implemented in OpenFOAM and validated on several benchmark problems. RANS simulations for the circulation control airfoil were run at two jet momentum coefficients: $C_j = 0.044$ and $C_j = 0.12$ using an incompressible solver. The boundary conditions used are: freestream velocity of $34m/s$ is specified at the inflow and, at the plenum inlet, the velocities given in the table 1 are used. The walls of the plenum and the airfoil surface are treated as no-slip boundaries. Slip wall boundary condition is imposed on the tunnel walls. The simulations are run to the atmospheric pressure at the exit.

<table>
<thead>
<tr>
<th>$C_j$ (approx.)</th>
<th>$U_{j, \text{max}}$</th>
<th>$U_{\text{inlet}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.044</td>
<td>130</td>
<td>4.203</td>
</tr>
<tr>
<td>0.12</td>
<td>210</td>
<td>6.924</td>
</tr>
</tbody>
</table>

Table 1. Flow conditions

Figure 2 presents streamtraces with different models and the variation of pressure coefficient over the airfoil surface at $C_j = 0.044$. The character of the separation bubble is found to be different with different
curvature corrections, as is also observed by Rumsey and Nishino. The pressure coefficient predictions are improved by accounting for the curvature. Both the curvature correction models give similar results for this case.

As shown in figure 3, the velocity profiles predicted by the models are not in agreement with the LES data. The peak velocity in the wall-jet region is slightly improved with curvature corrections. The lift coefficient calculated with different models is given in table 2. SST k-ω predicts higher circulation and hence higher lift. When a curvature correction is used, the location at which the Coanda jet separates from the airfoil surface moved up, resulting in lower circulation and hence lower lift coefficient. But, the effect is small.

At $C_j = 0.12$, the streamtraces plotted in figure 4 clearly shows that the use of curvature correction moves the separation location slightly up over the Coanda surface. The pressure coefficient near the trailing edge is accurate compared to the LES data. As with the lower jet momentum coefficient case, the near-wall peak in the velocity profiles improved significantly by accounting for the curvature (see figure 5). The lift coefficient given in table 3 is consistent with the above observations.

Figure 2. Case: $C_j = 0.044$, Separation characteristics on the Coanda surface at the trailing edge of the airfoil

(a) SST k-ω
(b) SST k-ω with bifurcation model
(c) SST k-ω with modified coefficients model
(d) Pressure coefficient variation along the airfoil surface
Figure 3. Case: $C_\theta = 0.044$, Radial velocity profiles comparison with the LES data
<table>
<thead>
<tr>
<th>Model</th>
<th>Lift coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>LES (Nishino et al.\textsuperscript{13})</td>
<td>1.36</td>
</tr>
<tr>
<td>SST k-ω</td>
<td>1.71</td>
</tr>
<tr>
<td>Bifurcation model</td>
<td>1.66</td>
</tr>
<tr>
<td>Modified coefficients model</td>
<td>1.62</td>
</tr>
</tbody>
</table>

Table 2. Lift coefficient predictions at $C_j = 0.044$

Figure 4. Case: $C_j = 0.12$, Separation characteristics on the Coanda surface at the trailing edge of the airfoil
Figure 5. Case: $C_l = 0.12$, Radial velocity profiles comparison with the LES data
<table>
<thead>
<tr>
<th>Model</th>
<th>Lift coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>LES (Nishino et al.\textsuperscript{13})</td>
<td>3.5</td>
</tr>
<tr>
<td>SST $k-\omega$</td>
<td>4.40</td>
</tr>
<tr>
<td>Bifurcation model</td>
<td>3.91</td>
</tr>
<tr>
<td>Modified coefficients model</td>
<td>4.13</td>
</tr>
</tbody>
</table>

Table 3. Lift coefficient predictions at $C_j = 0.12$

IV. Concluding remarks

From the results reported in this paper, it can be concluded that the streamline curvature plays a definite role in predicting the aerodynamic characteristics of circulation control airfoil. The curvature effects are more pronounced at higher jet momentum coefficients.

The two approaches of streamline curvature modeling did not show any significant differences. A consistent improvement over the base turbulence model means that these curvature correction models are reliable. However, the discrepancies observed in the velocity, lift coefficient predictions between RANS and LES need to be further investigated.

Acknowledgments

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References


