Reduced order modeling of choked blade-rows in axial flow compressors

Sunil K. Arolla, Giridhar Jothiprasad, Trevor H. Wood, Andrew Breeze-Stringfellow

1 Turbo-machinery Design System Division, GE Aircraft Engines, Bangalore, India, sunil.arolla@ae.ge.com
2 Fluid Mechanics Laboratory, GE Global Research Centre, Niskayuna, U.S., jothipra@research.ge.com
3 Computational Heat Transfer and Aeroacoustics Laboratory, GE Global Research Centre, Niskayuna, U.S., woodth@research.ge.com
4 Compressor and Turbine Aero division, GE Aircraft Engines, Cincinnati, U.S., andy.breeze-stringfellow@ae.ge.com

Abstract

Reduced-order modeling of turbo-machinery performance is an important part of the engine design cycle from preliminary design to experimental test evaluation. Detailed aerodynamic design carried out using high fidelity 3D CFD simulations is limited in scope to only a selection of critical operating conditions, owing to the high computational expense associated with the analysis. To estimate turbo-machinery performance over a full range of operating conditions in a computationally expedient manner, well calibrated sub-system modeling is very useful in pushing toward a more robust, optimal design of the overall component.

In this research, models to identify blade-row choke and predict the blade-row behavior beyond choke are developed and incorporated into a pitch-line prediction model for off-design performance of axial compressors. This model characterizes the behavior of each blade-row by flow conditions at the mid-span of the leading and trailing edges using various aerodynamic models for flow loss, deviation and blockage. Blade-row choke occurs when the mass flow rate through the blade-row cannot be increased further by a reduction in downstream static pressure. The developed choke model uses the 2D blade-to-blade flow at the pitch-line to characterize the choking of the 3D blade-row.

Three different types of blade passage choking are identified based on the inlet Mach number to the blade-row. The choke point is specified by modeling the minimum inlet flow angle to the axial direction at which the passage chokes as a function of inlet Mach number. In all three types of choke, the flow is sonic (or supersonic for unique incidence) across the entire passage at a particular axial location. Hence, no disturbances can propagate upstream of this axial location and any reductions in downstream static pressure increases the in-passage shock strength in the last choked blade-row without increasing the mass flow rate. This allows the increased loss due to a stronger shock to be modeled in a unified fashion for all mechanisms of choke. The model is validated on a single-stage commercial fan testcase. Comparisons of the model predictions for the commercial fan with experimental data have been encouraging.

Introduction

With the continuous reduction in the aircraft engine design cycle time, there is an acute need to quickly evaluate candidate compressor designs. The high computational expense of turbo-machinery CFD restricts its application to a few critical operating points, while the high cost of experimental rig testing restricts testing to a few candidate designs. Hence, reduced order modeling of compressors over the full range of operating conditions is a valuable tool in the optimal design of compressors. In this research, a pitch-line prediction model for off-design performance of axial compressors is used to characterize the behavior of each blade-row by flow conditions at the mid-span of the leading and trailing edges using various aerodynamic models for flow loss, deviation and blockage.

The maximum flow swallowing capacity of the compressor is a quantity of interest to the designers. This problem can be analyzed in a blade-row-by-blade-row basis as a single blade-row determines the limiting choke flow through the entire compressor. In this paper, simplified models based on the pitch-line geometry of each blade-row are used to identify the limiting choke flow through each blade-row. These blade-row models are then integrated into the pitch-line prediction model for the compressor mentioned above to determine maximum flow through the entire compressor. Once any blade-row of the compressor chokes, a drop in the static pressure at the compressor exit is not accompanied by an increase in mass flow but by an increase in losses in the choked blade-rows. In this paper, the
losses introduced due to blade-row choking are also modeled to allow the computation of compressor characteristics with choked blade-rows.

The choke point is specified by modeling the minimum inlet flow angle with respect to the axial direction at which the passage chokes as a function of inlet Mach number. This minimum inlet flow angle not only depends on the blade passage geometry such as the distributions of thickness and camber (particularly near the leading edge), the inlet and stagger angle but also on the three-dimensional features introduced by the radial variations of the blade. However, the purpose of this paper is to develop and evaluate a reduced order model that can be integrated into the pitch-line prediction model for off-design compressor performance. Hence a number of the three-dimensional features such as shock inclination and radial migration of streamlines are neglected and only a subset of the geometry parameters at the pitch-line is used in the model development.

Three different types of blade passage choking are identified based on the inlet Mach number to the blade-row. In Section 2, modeling for subsonic inlet flow is discussed. When the relative flow at inlet is supersonic, the shock at the blade leading edge can either be detached or attached leading to different analyses presented in sections 3 and 4 respectively. Finally, the model predictions for a single stage fan are presented in Section 5.

Subsonic choke model

When the inlet flow is subsonic, analysis outlined in Cumpsty is used to predict blade-passage choke. When the relative inlet flow angle (measured from axial) is reduced, the mass flow into the blade passage increases. The minimum inlet flow angle at which the blade-row chokes is to be determined as a function of the inlet Mach number. Figure 1 shows the schematic of a choked blade passage with uniform subsonic inlet flow. The flow is assumed uniform across the passage and hence the sonic line at the throat is exactly straight. A control volume extending between the uniform conditions upstream and the sonic line across the throat is used for further analysis. As the flow is subsonic inside the control volume, there are no shocks and the flow is treated as isentropic. With these idealizations, mass conservation and isentropic relations can be used to derive the following relation between minimum inlet flow angle at choke ($\alpha_1$) and inlet Mach number ($M_1$):

$$\frac{b}{s \cos(\alpha_1)} = M_1 \left[ \frac{1 + \frac{\gamma-1}{2} \left( \frac{1}{1 + \frac{\gamma-1}{2} M_1^2} \right)^{\frac{2(\gamma-1)}{\gamma+1}}}{\frac{\gamma+1}{2(\gamma-1)}} \right]$$

where $b$ is the throat width, $s$ is the staggered inlet gap measured in axial direction, $\gamma$ is the specific heat ratio. Figure 2, referred to as the choke curve, is the variation of the minimum inlet flow angle at which blade-row chokes with inlet Mach number. The portion of the choke curve spanning subsonic inlet Mach numbers is obtained from the above equation.

In the case of subsonic inlet flow, any shocks, if they exist, would be downstream of the sonic line at the throat. The backpressure at the exit of the blade-row determines the location and strength of this shock system. When the back-pressure to the blade-row is reduced, the shock system downstream of the throat increases in strength to match the required exit pressure. The focus of this reduced order modeling effort is compressor performance prediction, therefore details of the shock location, etc., are not modeled whereas the shock losses and its effect on the
compressor performance accounted for. At present, the changes in deviation that might be introduced due to shock-induced flow separation are also not modeled.

In this paper, the additional shock losses introduced when the back-pressure is lowered beyond that at blade-row choke are referred to as choke losses. Since this model is integrated into a reduced-order model for the compressor as a whole, only the back-pressure at the compressor exit is specified and not the back pressure at the exit to the choked blade-row. To determine the choke losses, the first step is to identify the last (i.e., furthest downstream) choked blade-row. Since the flow upstream of the last choked blade-row remains unaltered by changes to the compressor exit pressure, additional choke losses are introduced only in this last choked blade-row. These choke losses are modified until either the specified compressor back-pressure is matched or another downstream blade-row starts to choke. If a downstream blade-row chokes, the losses in this newly choked blade-row are modified to match the specified compressor exit pressure. This algorithm allows us to determine the choke losses in the different blade-rows to match a given compressor exit pressure. Robust numerical algorithms are used to implement the above procedure for determining choke losses.

**Supersonic inlet flow with detached shocks:**

![Supersonic inlet flow - detached shock](image)

For blades with supersonic inlet flow, importance is given to the inlet region of the blade that is ahead of a line from the leading edge drawn normal to the adjacent blade suction surface. This region produces most of the pressure rise through shock compression as well as defines the maximum flow through the blade passage. Further, the shocks at the leading edge can either be attached or detached. In this section, a simple analysis by Freeman and Cumpsty, shown schematically in figure 3, is used for modeling detached shocks. A control volume is chosen to extend from the uniform inlet to the throat where there is maximum thickness. Flow enters at Mach number $M_1$ and at relative angle $\alpha_1$ to the axial direction. Most supersonic blades are lightly cambered in the inlet region, hence a single parameter, stagger angle $\chi_1$, suffices to describe the blade there. Throat width $b$ of the blade-row is calculated from the maximum thickness and stagger angle as $s \cos \chi_1 - t$ where $t$ is the maximum blade thickness. The assumption of isentropic flow used in the subsonic inlet flow analysis within the control volume is no longer valid due to the shock(s) near the leading edge. When the blade-row chokes the sonic line extends across the throat. Simplifying the conservation equations for mass, momentum and enthalpy, yields the following equation relating the inlet Mach number $M_1$ to the inlet flow angle at choke:

$$\left[1 + \frac{\gamma - 1}{2} M_1^2 \right]^{-1/2} \left( \cos \chi_1 / \cos \alpha_1 + \gamma M_1^2 \cos(\alpha_1 - \chi_1) \right) = \left[1 + \frac{\gamma - 1}{2} \right]^{-1/2} \frac{(1 + \beta)s \cos \chi_1}{b}$$

Figure 2 shows the choke curve computed using the above equation for detached shocks with sample geometry parameters $t/c = 0.05$ and $s/c = 1.0$ and blade angle = $65^\circ$ for illustration purposes. Finally, the computation of choke losses beyond detached shock choke is the same as that for subsonic inlet flow. This is because, in either case a sonic line exists at the throat and only the flow downstream of the throat is modified by changes to the back-pressure.

**Supersonic inlet flow with attached shocks (Unique-incidence):**

The last type of blade passage choke is known as “unique incidence” flow where the supersonic inlet flow results in shocks that are attached to the blade leading edge. Figure 5 shows an idealized “unique incidence” flow condition in which the leading edge is assumed to be so sharp that the shock is attached. The passage shock produces much of the pressure rise while the bow shock upstream of the blade is weak and does not produce much compression. Any curvature of the blade suction surface in the inlet region, where the flow is supersonic, brings about a Prandtl-Meyer expansion of the flow leading to a higher Mach number at the inlet section to the blade and therefore lower choking mass flow.
Levine\textsuperscript{3} developed a simple analysis that captures the important aspects of the unique incidence flow condition. The leading edge is assumed to be sufficiently sharp that the passage shock is attached. Hence, there exists a unique characteristic a-b (shown in Figure 4) that leaves the suction surface and intersects with the leading edge of the adjacent blade. Further, the entropy rise in the weak bow shock upstream of the blade is neglected and the bow shock is treated as an isentropic compression. Due to this reason, the Prandtl-Meyer relations can be applied between the conditions far upstream, subscript 1, and along line a-b, subscript e, to give

$$\alpha_1 + \nu(M_1) = \alpha_e + \nu(M_e)$$  \hspace{1cm} (1)

where $\alpha$ is the relative flow direction measured from axial and $\nu$ is the Prandtl-Meyer function given by

$$\nu(M) = \sqrt{\frac{\gamma-1}{\gamma+1}} \tan^{-1} \left( \frac{\gamma-1}{\gamma+1} (M^2 - 1) \right) - \tan^{-1} \sqrt{M^2 - 1}$$

The other condition that is satisfied between far upstream and the line a-b is the mass continuity equation for an isentropic flow:

$$F(M_1)A_1 = F(M_e)A_e$$  \hspace{1cm} (2)

where $A_1 = s \cos \alpha_1$ is the incoming stream-tube width, $A_e$ is the stream-tube width across a-b and $F$ is the flow function given by

$$F = \frac{m}{Ap_0} = \frac{\gamma}{\sqrt{\gamma-1}} M \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{-\gamma/2(\gamma-1)}$$

For a given blade geometry $\alpha_e$ and $A_e$ are fixed but an iterative process is required to determine the exact location of a on the suction surface. However, to further simplify the analysis, $\alpha_e$ is assumed to be the inlet stagger angle and $A_e$ is taken to be the throat width in the following analysis. Hence, the three unknowns, $\alpha_1$, $M_1$, and $M_e$ must satisfy two independent non-linear equations (1-2). This implies that for a specified inlet Mach number there exists a unique inlet flow angle at which the two independent equations are satisfied. Also if this unique incidence cannot be changed, neither can the mass flow and hence the flow is choked.

Figure 2 shows the choke curve computed for the unique incidence condition for sample geometry parameters $t/c = 0.05$ and $s/c = 1.0$ and blade angle = 65°. Finally, the computation of choke losses beyond unique incidence choke is the same as before. This is because the flow is supersonic at line a-b and any changes in back-pressure can affect only the flow downstream of line a-b. Hence, the choke losses are computed to match the specified back-pressure using the same algorithm as before.

**Results:**

In the reduced-order modeling approach, the total losses are divided into different loss components that can be modeled. The total pressure loss coefficient, defined as follows, is used:

$$\text{Pressure loss coef.} = \frac{\text{Stagnation pressure loss}}{\text{Inlet dynamic head}}$$

The choke loss found using the proposed approach is added to the total losses and used in blade-row aero-thermal calculations to get the overall performance of the compressor.
The choke loss model implemented is validated on a commercial fan test case. Because of high relative Mach numbers involved, fans often operate choked at high speeds. Figure 3 shows the characteristic map (pressure ratio vs. corrected flow) obtained from the aero-thermal calculations with and without choke-loss model and from experimental data for a commercial fan test case. It can be observed that choke limit curve can be calibrated to identify the choking mass flow over a range of high speed-lines. Further, with the added choke losses, the performance can be predicted in the vertical part of the speed-line. Figure 4 shows that for a given total pressure ratio the model under-predicts the adiabatic efficiency in the choked portion of the speed-line. This seems to suggest that the model is over-predicting the work done by the blade-row and hence can possibly be improved by including additional deviation modeling for choked blade-rows. Improved deviation modeling is one of the areas being pursued for future research.

**Summary:**

The proposed model is validated on a commercial fan testcase and the results show that the choking mass flow as well as performance in the choked portion of the speed-line can be predicted. Future investigations will be focused on validating the loss model on a high-pressure compressor where the possibility exists for multiple choked blade rows. Furthermore, additional modeling efforts will be undertaken to improve the work and efficiency predictions on the choke side.

**References:**